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Tangential Representations at Fixed Points

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1. Basic problems

Let G be a finite group throughout this paper. We mean by a (real) *G-module* a real G-representation (space) of finite dimension. Let $\mathcal{S}(G)$ denote the set of all subgroups of G and let $\mathcal{P}(G)$ denote the subset of $\mathcal{S}(G)$ consisting of all subgroups of prime power order. Unless otherwise stated, M will stand for a (smooth) G-manifold. S. Cappell-J. Shaneson referred the next problem to a basic problem on Algebraic and Differential Topology.

Problem (Basic Problem A). Let x, $y \in M^G$. How similar is a neighborhood of x to that of y as G-spaces?

If $x \in M^G$, then we can regard the tangent space $T_x(M)$ at x in M as a G-module. Thus the problem above is equivalent to ask

Problem (Basic Problem B). How similar is $T_x(M)$ to $T_y(M)$ as G-modules?

A specific case of the problem was posed by P. A. Smith.

Problem (Smith Problem). If Σ is a homotopy sphere with exactly two fixed points x and y, then is $T_x(\Sigma)$ isomorphic to $T_y(\Sigma)$ as G-modules?

We would like to study this problem in a slightly generalized form. Now let $\mathfrak{A}(2)$ denote the family of all (smooth) G-actions on manifolds with exactly 2 fixed points and let $\mathfrak{X} \subset \mathfrak{A}(2)$. We say that G-modules V and W are *$\mathfrak{X}$-related*, and write $V \sim_{\mathfrak{X}} W$, if there exists a smooth G-action on $M \in \mathfrak{X}$ such that $M^G = \{a, b\}$, $T_a(M) \cong_G V$ and $T_b(M) \cong_G W$. Let $\mathrm{RO}(G)$ denote the real representation ring of G. We define the *$\mathfrak{X}$-relation set* $\mathrm{RO}(G, \mathfrak{X})$ *of* G by

$$\mathrm{RO}(G, \mathfrak{X})=\{[V] - [W] \in \mathrm{RO}(G) \mid V \sim_{\mathfrak{X}} W\}$$

Problem (Basic Problem C). Describe $\mathrm{RO}(G, \mathfrak{X})$ in terms of Algebra (or Representation Theory)

We say that a G-action on a disk D has a *linear boundary action* if the boundary ∂D is G-diffeomorphic to the unit sphere $S(V)$ for some G-module V. A G-action on a homotopy sphere Σ is called a *G-semilinear sphere* if Σ^H is a homotopy sphere for each $H \le G$. G-modules V and W are called *$\mathcal{P}$-matched* if $\mathrm{res}^G_P V \cong_P \mathrm{res}^G_P W$ for all $P \in \mathcal{P}(G)$.

We will discuss Basic Problem C for the following subfamilies of $\mathfrak{A}(2)$.

$$\begin{aligned}
\mathfrak{E} &= \{G\text{-actions on Euclidean spaces} \in \mathfrak{A}(2)\} \\
\mathfrak{D} &= \{G\text{-actions on disks} \in \mathfrak{A}(2)\} \\
\mathfrak{D}_{\partial\text{-lin}} &= \{G\text{-actions on disks with linear boundary action} \in \mathfrak{A}(2)\} \\
\mathfrak{S} &= \{G\text{-actions on homotopy spheres} \in \mathfrak{A}(2)\} \\
\mathfrak{S}_{\text{s-free}} &= \{\text{semi free actions} \in \mathfrak{S}\} \\
\mathfrak{S}_{CS} &= \{\Sigma \in \mathfrak{S} \text{ such that } |\Sigma^H| = 2 \text{ or } \Sigma^H \text{ is connected } (\forall\ H \le G)\ \} \\
\mathfrak{S}_{\text{s-lin}} &= \{G\text{-semilinear spheres} \in \mathfrak{A}(2)\} \\
\mathrm{p}\mathfrak{S} &= \{\Sigma \in \mathfrak{S}\ (\Sigma^G = \{x, y\}) \text{ such that } T_x(\Sigma) \text{ and } T_y(\Sigma) \text{ are } \mathcal{P}\text{-matched}\}
\end{aligned}$$

With this notation, the Smith Problem is equivalent to ask whether $\mathrm{RO}(G, \mathfrak{S}) = 0$ or not.

Here we may remark the following.

Theorem (G. E. Bredon [2]). *Let $G = C_n$ with $n = p^a$ and $\Sigma \in \mathfrak{S}$ with $\dim \Sigma = 2k$ and x, $y \in \Sigma^G$. Then $T_x(\Sigma) - T_y(\Sigma)$ is divisible by p^h in $\mathrm{RO}(G)$, where $h = \left[\dfrac{pk - n}{pn - n}\right]$.*

By T. Petrie (e.g. [24]), the theorem above implies that if $\dim \Sigma \gg n$ then $T_x(\Sigma) \cong_G T_y(\Sigma)$. Thus, in the case $G = C_n$ with $n = 2^a \ge 8$, the set $\mathrm{RO}(G, \mathfrak{S})$ is not additively closed.

2. Preliminary

Let $\mathcal{H}$ be a set of subgroups of G. G-modules V and W are called *$\mathcal{H}$-matched* if $\mathrm{res}^G_H V \cong_H \mathrm{res}^G_H W$ for all $H \in \mathcal{H}$. A G-module V is called *$\mathcal{H}$-free* if $V^H = 0$ holds for any $H \in \mathcal{H}$. For $M \subset \mathrm{RO}(G)$, and $\mathcal{H}$, $\mathcal{K} \subset \mathcal{S}(G)$, we define

$$M_{\mathcal{H}} = \{V - W \in M \mid V \text{ and } W \text{ are } \mathcal{H}\text{-matched}\}$$

$$M^{\mathcal{K}} = \{V - W \in M \mid V,\, W \text{ are } \mathcal{K}\text{-free}\}$$

$$M^{\mathcal{K}}_{\mathcal{H}} = M_{\mathcal{H}} \cap M^{\mathcal{K}}.$$

By Definition, we have $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = \mathrm{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}$.

In some other papers, V and W are called *Smith equivalent* if $V \sim_{\mathfrak{S}} W$; V and W are called *s-Smith equivalent* if $V \sim_{\mathfrak{S}_{\text{s-lin}}} W$; V and W are called *primary Smith equivalent* if $V \sim_{\mathfrak{p}\mathfrak{S}} W$. The set $Sm(G) = \mathrm{RO}(G, \mathfrak{S})$ was usually called the *Smith set* and the set $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S})$ *primary Smith set*. By definition, $Sm(G)_{\mathcal{P}(G)} = \mathrm{RO}(G, \mathfrak{p}\mathfrak{S})$.

A finite group G is called a *mod $\mathcal{P}$ cyclic group* if there exists a normal subgroup P of G such that P is of prime power order and G/P is cyclic. G is called a *mod $\mathcal{P}$ hyperelementary group* if there exists a normal series $P \trianglelefteq H \trianglelefteq G$ such that P and G/H are of prime power order and H/P is cyclic. G is called an *Oliver group* if G is not a mod $\mathcal{P}$ hyperelementary group. Thus G is an Oliver group if and only if G admits a G-action on a disk without fixed points.

Let p be a prime. Let $G^{\{p\}}$ denote the smallest normal subgroup H of G such that G/H has the order of a p-power. We refer $G^{\{p\}}$ to the *Dress subgroup of type p*. Let G^{nil} denote the smallest normal subgroup H of G with nilpotent G/H. It follows that

$$G^{nil} = \bigcap_q G^{\{q\}}.$$

Let us adopt the following notation.

$$\mathcal{PC}(G) = \{\text{mod-}\mathcal{P} \text{ cyclic subgroups of } G\}$$

$$\mathcal{L}(G) = \{L \in \mathcal{S}(G) \mid L \supset G^{\{p\}} \text{ for some prime } p\}$$

$$\mathcal{M}(G) = \mathcal{S}(G) \smallsetminus \mathcal{L}(G)$$

3. Classical results (until 1996)

There are various affirmative answers to the Smith Problem. It is easy to see that if $V \sim_{\mathfrak{S}} W$ then $\mathrm{res}^G_P V \cong_P \mathrm{res}^G_P W$ for all $P \in \mathcal{P}(G)$ with $|P||4$. By Atiyah-Bott and Milnor, $V \sim_{\mathfrak{S}_{\text{s-free}}} W$ implies $V \cong_G W$. Sanchez showed that $V \sim_{\mathfrak{S}} W$ imples $\mathrm{Res}^G_P V \cong_P \mathrm{Res}^G_P W$ for any P of odd-prime-power order.

To the contrary, there are negative answers to the Smith Problem. T. Petrie showed that if G is an odd-order abelian group containing $C_{pqrs} \times C_{pqrs}$, where p, q, r, s are distinct odd primes, then $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \neq 0$. In addtion, Cappell-Shaneson showed that if $G = C_{4n}$ with $n \geq 2$ then $\mathrm{RO}(G, \mathfrak{S}_{CS}) \neq 0$.

Here we also recall classical results concerned with $\sim_{\mathfrak{E}}$ and $\sim_{\mathfrak{D}}$. By Petrie, if G is an odd-order abelian group, then $\mathrm{RO}(G, \mathfrak{D})^{\mathcal{L}(G)} = \mathrm{RO}(G)^{\mathcal{L}(G)}_{\mathcal{P}(G)}$. R. Oliver showed that if G is not of prime power order, then $\mathrm{RO}(G, \mathfrak{E}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)}$; if G is an Oliver group, then $\mathrm{RO}(G, \mathfrak{D}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)}$.

4. Dimension Conditions on G-Modules

In order to apply an equivariant surgery theory to a G-manifold M, we require certain properties for M^H, where $H \in \mathcal{S}(G)$. If $V = T_x(M)$ with $x \in M^G$, then $\dim V^H$ is equal to the dimenison of the connected component of M^H containing the point x.

Let V be a G-module.

(1) We say that V satisfies the *strong gap condition* if $\dim V^P > 2\dim V^H + 2$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$.

(2) We say that V satisfies the *gap condition* if $\dim V^P > 2\dim V^H$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$.

(3) We say that V satisfies the *weak gap condition* if the next dimension condition:

(Dim) $\dim V^P \geq 2\dim V^H$ for all $P < H \leq G$ with $P \in \mathcal{P}(G)$

is satisfied and V satisfies the orientation condition:

(Ori) $g : V^H \to V^H$ preserves orientation for any $g \in N_G(P) \cap N_G(H)$ such that $P \in \mathcal{P}(G)$, $P < H \leq G$ and $\dim V^P = 2\dim V^H$.

A finite group G is called a *gap group* if there exists a G-module V such that V is $\mathcal{L}(G)$-free and satisfies the gap condition.

5. Laitinen's Conjecture

E. Laitinen and K. Pawałowski were interested in determining the set $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S})$, namely $\mathrm{RO}(G, \mathfrak{S})_{\mathcal{P}(G)}$.

Conjecture (E. Laitinen). Let G be an Oliver group. Then $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \neq 0$ holds if and only if $\mathrm{RO}(G, \mathfrak{D}) \neq 0$.

For $g \in G$, let (g) denote the conjugacy class $\{aga^{-1} \in G \mid a \in G\}$, and let $(g)^{\pm}$ denote the *real conjugacy class* $(g) \cup (g^{-1})$. Then a_G stands for the number of all real conjugacy classes $(g)^{\pm}$ such that $g \in G$ is not of prime power order. If G is an Oliver group, since $\mathrm{RO}(G, \mathfrak{D}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)}$, we obtain $\mathrm{rank}\mathrm{RO}(G, \mathfrak{D}) = a_G - 1$.

Theorem (E. Laitinen-K. Pawałowski, K. Pawałowski-R. Solomon, M. Morimoto). *Laitinen's Conjecture has been studied and is affirmative for Oliver gap groups G satisfying one of the following conditions.*

(1) *G is a perfect group* [9].

(2) *G is a nonsolvable group:*

- *Case $G \not\cong P\Sigma L(2, 27)$:* [20].
- *Case $G = P\Sigma L(2, 27)$:* $\mathrm{RO}(G, \mathfrak{S}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)} \cong \mathbb{Z}$ [12].

(3) *G has a normal subgroup N such that $G/N \cong C_{pq}$ with distinct odd primes p, q* [20].

(4) *G is of odd order* [20].

Let $SG(m, n)$ denote the nth small group of order m given by the computer software GAP [5].

Theorem (A. Koto-M. Morimoto-Y. Qi, M. Morimoto, T. Sumi). *Laitinen's Conjecture fails and* $\mathrm{RO}(G, \mathfrak{S}) = 0$ *for Oliver groups G satisfying one of the following conditions.*

(1) $G = \mathrm{Aut}(A_6)$ *(nongap group, $G/G^{nil} = C_2 \times C_2$)* [14].

(2) $G = SG(72, 44)$ (*gap group*, $G/G^{nil} = C_6$) [28].

(3) $G = SG(288, 1025)$ (*gap group*, $G/G^{nil} = C_6$) [28].

(4) $G = SG(432, 734)$ (*nongap group*, $G/G^{nil} = C_2$) [28].

(5) $G = SG(576, 8654)$ (*nongap group*, $G/G^{nil} = C_2 \times C_2$) [28].

(6) $G = SG(1176, 220)$ (*gap group*, $G/G^{nil} = C_3$) [7].

(7) $G = SG(1176, 221)$ (*gap group*, $G/G^{nil} = C_3$) [7].

6. Determination of $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S})$

Throughout this section, let G be an Oliver group.

Theorem (K. Pawałowski-R. Solomon [20]). *Let G be an Oliver group.*

(1) *If G is a gap group, then* $\mathrm{RO}(G, \mathfrak{S})^{\mathcal{L}(G)}_{\mathcal{P}(G)} = \mathrm{RO}(G)^{\mathcal{L}(G)}_{\mathcal{P}(G)}$.

(2) *If G is either an Oliver group of odd order or a nonsolvable group $\not\cong \mathrm{Aut}(A_6)$, $P\Sigma L(2, 27)$ and if $a_G \geq 2$, then* $\mathrm{RO}(G)^{\mathcal{L}(G)}_{\mathcal{P}(G)} \neq 0$.

Let us define the following subsets of $\mathrm{RO}(G)$.

$\mathrm{RO}[\mathcal{H}^{\mathcal{L}}](G) = \{V - W \in \mathrm{RO}(G) \mid V, W$ are $\mathcal{L}(G)$-free and satisfy (Dim)$\}$

$\mathrm{RO}[\mathcal{W}^{\mathcal{L}}](G) = \{V - W \in \mathrm{RO}(G) \mid V, W$ are $\mathcal{L}(G)$-free and satisfy (Dim), (Ori)$\}$

where (Dim) and (Ori) stand for the dimension condition and the orientation condition, respecively, appearing in the weak gap condition (see Section 4).

By definition,

$$2 \cdot \mathrm{RO}[\mathcal{H}^{\mathcal{L}}](G) \subset \mathrm{RO}[\mathcal{W}^{\mathcal{L}}](G) \subset \mathrm{RO}[\mathcal{H}^{\mathcal{L}}](G).$$

If G is a gap group, then $\mathrm{RO}[\mathcal{W}^{\mathcal{L}}](G) = \mathrm{RO}(G)^{\mathcal{L}(G)}$.

By the Deleting-Inserting Theorem by M. Morimoto stated in [16, Appendix], we obtain the next basic theorem.

Theorem 6.1. *If G is an Oliver group, then*

$$\mathrm{RO}[\mathcal{W}^{\mathcal{L}}](G)_{\mathcal{P}(G)} \subset \mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \cap \mathrm{RO}(G, \mathfrak{D}_{\partial\text{-}lin}).$$

Corollary 6.2. *If G is an Oliver group with* $\mathrm{RO}[\mathcal{H}^{\mathcal{L}}](G)_{\mathcal{P}(G)} \neq 0$, *then* $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) \neq 0$.

X.M. Ju applied the theorem above and obtained the next result.

Theorem (X.M. Ju). *Let $X_2 = C_2 \times \cdots \times C_2$ be the n-fold cartesian product of C_2, where $n \geq 1$. Then $G = S_5 \times X_2$ is a nongap Oliver group,*

$$\mathrm{RO}(G, \mathfrak{S}) = \mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = \mathrm{RO}(G)^{\{A_5\}}_{\mathcal{P}(G)}$$

and

$$\mathrm{rank}_{\mathbb{Z}} \mathrm{RO}(G)^{\{A_5\}}_{\mathcal{P}(G)} = 2^n - 1.$$

Lemma 6.3 ([7]). *Let G be a finite group not of prime power order, N a normal subgroup of G, N_2 a Sylow 2-subgroup of N.*

(1) *If $G/N \cong C_2$ and $V \sim_{\mathfrak{S}} W$, then $V^N = 0 = W^N$ or $\mathrm{res}^G_N V \cong_N \mathrm{res}^G_N W$.*

(2) *If $G/N \cong C_p$ with p odd prime, N_2 is normal in N, and $V \sim_{\mathfrak{S}} W$ then $V^N = 0 = W^N$ or $\mathrm{res}^G_N V \cong_N \mathrm{res}^G_N W$.*

Lemma 6.4 ([7]). *Let G be a finite group not of prime power order and G_2 a Sylow 2-subgroup of G.*

(1) *If $G/G^{\{2\}} \cong C_2 \times \cdots \times C_2$, then $\mathrm{RO}(G, \mathfrak{S}) \subset \mathrm{RO}(G)^{\{G^{\{2\}}\}}$.*

(2) *If G_2 is normal in G and $G/G^{\{3\}} \cong C_3 \times \cdots \times C_3$, then $\mathrm{RO}(G, \mathfrak{S}) \subset \mathrm{RO}(G)^{\{G^{\{3\}}\}}$.*

Theorem 6.5 ([12]). *Let G be either $SG(864, 2666)$ or $SG(864, 4666)$. Then G is an Oliver group with $G/G^{nil} \cong C_3$ and*

$$\mathrm{RO}(G, \mathfrak{S}) = \mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = \mathrm{RO}(G)^{\{G\}}_{\mathcal{P}(G)} \cong \mathbb{Z}.$$

Let G be a finite Oliver group of order ≤ 2000. T. Sumi (2006) tried to see whether $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = 0$ or not. Putting his computation together with our results, we can determine whether $\mathrm{RO}(G, \mathfrak{p}\mathfrak{S}) = 0$ or not for G except ones in the next list:

$G(m,n)$	a_G	gap?	G/G^{nil}
$G(864, 4663)$	3	No	C_8
$G(864, 4672)$	5	Yes	$Q_8 \times C_3$
$G(1152, 155470)$	2	Yes	C_6
$G(1152, 157859)$	2	Yes	C_6

List 1

7. Conjectures

We have several conjectures related to the Smith Problem which are not yet proved.

Conjecture (S. E. Cappell-J. L. Shaneson). If $V \sim_{\mathfrak{S}_{CS}} W$ and the actions on V and W are pseudofree, then $V \simeq_G W$ (G-homeomorphic).

Conjecture 7.1. If G is an Oliver group with $\mathrm{RO}(G)^{\mathcal{L}(G)}_{\mathcal{P}(G)} \neq 0$, then $\mathrm{RO}(G,\mathfrak{S})^{\mathcal{L}(G)}_{\mathcal{P}(G)} \neq 0$.

Let c_G denote the number of the conjugacy classes (C) of cyclic subgroup C of G such that the order of C is not of prime power order. Let Γ denote the Galois group $Gal(\mathbb{Q}(\zeta))$, where $\zeta = \exp\left(\dfrac{2\pi\sqrt{-1}}{|G|}\right)$

Conjecture 7.2. If G is an Oliver group with $c_G \geq 2$, then $\mathrm{RO}(G,\mathfrak{pS})^{\Gamma} \neq 0$

Conjecture 7.3. If G is an Oliver group, then $\mathrm{RO}(G,\mathfrak{pS}) \subset \mathrm{RO}(G,\mathfrak{D}_{\partial\text{-lin}})$.

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